ELECTRIC FIELD EXCITED IN AIR BY A PULSE OF GAMMA RAYS

Yu. A. Medvedev, B. M. Stepanov,
and G. V. Fedorovich

The characteristics of the electric field produced by air polarization during the passage of nonstationary Compton currents excited by a γ-ray pulse in low-density air are discussed. The influence of the field on the motion of the Compton electrons is taken into account. The amplitude and relaxation time of the field are evaluated. A polarization electric field is created through the action of a directed current of γ-rays in air because of the movement of the Compton electrons. This paper discusses the basic characteristics of the resultant field in low-density air. A similar problem was raised in [1], where the electromagnetic field excited by a nonstationary source of γ-radiation in the upper atmosphere was considered. In that case, the Compton-electron currents were specified and their magnitude was assumed to be proportional to the ratio between the gas kinetic ranges of Compton electron and γ-ray (this ratio is of the order of 0.01 and is indepenent of height). With an increase in electron range, however, the decelerating action of the resultant electric field on the motion of the Compton electron becomes important (eEl/ε is a criterion for the effect; E is the field intensity, and l and ε are the range and energy of the Compton electron).

1. We consider the question of the polarization field which is created in low-density air present in an intense flux of γ-radiation. The photon flux is assumed to be monoenergetic with an energy of the order of one MeV. Two cases are analyzed in the following: in one, the time dependence of the flux intensity is chosen to be in the form of a delta function; in the other, the time dependence is in the form of a single step function. Although each of the corresponding results separately does not completely characterize the nonlinear system under consideration, the combination of them is sufficient, as a rule, for evaluations in the majority of cases of practical interest. As in [2, 3], it is assumed that the Compton electrons move, on the average, in the direction of photon propagation. It is also assumed that the resultant field changes little in space over the range of a Compton electron. The growing conductivity of the air affects both the magnitude of the temporal behavior and the variation of the field, and the conductivity, in turn, depends on the magnitude of the field. We shall consider that the air conductivity σ is due to secondary electrons which are created by the Compton electrons and which disappear as the result of electron recombination at the rate α.

Designating the effective mobility of the secondary electrons by k and their density by n, we can write

$$\sigma = ekn$$

We consider the effect of the electric field E on the deceleration of a Compton electron. If the initial energy is ε_0, the energy loss during electron motion is described by

$$-\alpha\varepsilon / dr = f(\varepsilon) + eE \tag{1.1}$$

where $f(\alpha)$ is the ionization energy loss for an electron.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 3-8, July-August, 1970. Original article submitted December 30, 1969.

We obtain the effective electron range from Eq. (1.1) including ionization loss in the decelerating field:

$$l^* = \int_0^{\varepsilon_0} \frac{d\varepsilon}{f(\varepsilon) + eE}$$

In first approximation, one can set $f(\varepsilon) = \text{const}$ (more accurate considerations give a very similar result).

Then

$$l^* = \frac{l}{1 + eEl/\varepsilon_0} \tag{1.2}$$

The energy expended in ionization of the air is

$$\Delta\varepsilon = \int_0^{l^*} f[\varepsilon(r)]\, dr = \frac{\varepsilon_0}{1 + eEl/\varepsilon_0}$$

i.e., the effect of the field leads to a reduction of the energy expended in ionization by a factor of $(1 + eEl/\varepsilon_0)$. The number of secondary electrons produced is obviously also reduced by the same ratio:

$$\nu^* = \frac{\nu}{1 + eEl/\varepsilon_0} \tag{1.3}$$

where $\nu \approx 3 \cdot 10^4$ is the number of ion-electron pairs formed by the absorption of 1 MeV. Using an expression for the Compton current similar to that given in [2, 3] (in this case, one should use l^* and ν^* instead of l and ν):

$$j_k = e \frac{l^*}{l_\gamma} j_\gamma$$

and for the density of secondary electron sources using

$$q = \frac{\nu^*}{l_\gamma} j_\gamma$$

(l_γ is the γ-ray range, j_γ is the γ-ray flux at the point under consideration), we obtain a closed system of equations which describe the time variation of the field and the conductivity:

$$\partial E / \partial t = 4\pi\{j_k - \sigma E\}, \qquad \partial n / \partial t = q - \alpha n^2 \tag{1.4}$$

If we introduce the dimensionless variables

$$x = \frac{elE}{\varepsilon}, \quad y = \frac{\alpha l e^2}{\nu k \varepsilon} n, \quad \zeta = \frac{4\pi\nu k^2 \varepsilon}{\alpha l^2} t$$

the system (1.4) takes the form

$$\frac{dx}{d\zeta} = \frac{i(\zeta)}{1+x} - xy, \quad \frac{dy}{d\zeta} = \lambda\left\{\frac{i(\zeta)}{1+x} - y^2\right\} \tag{1.5}$$

$$\lambda \equiv \frac{\alpha}{4\pi e k}, \quad i = \frac{\alpha}{\nu l_\gamma} j_\gamma \left(\frac{el^2}{k\varepsilon}\right)^2$$

The initial conditions for x and y are zero.

2. We consider the variation in x and y for a pulsed source, i.e., for

$$i(\zeta) = i_0 \delta(\zeta)$$

We designate the values of x and y for $\zeta \to +0$ by x_0 and y_0, respectively. To determine the value of x_0, we multiply the first equation of the system (1.5) by $(1+x)$ and integrate over ζ from $\zeta = -0$ to $\zeta = \zeta_1$. We then have

$$\int_0^{\zeta_1} (1+x) \frac{dx}{d\zeta} d\zeta = x(\zeta_1) + \frac{x^2(\zeta_1)}{2} = i_0 - \int_0^{\zeta_1} xy(1+x)\, d\zeta$$

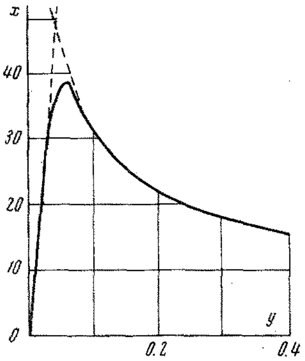

Fig. 1

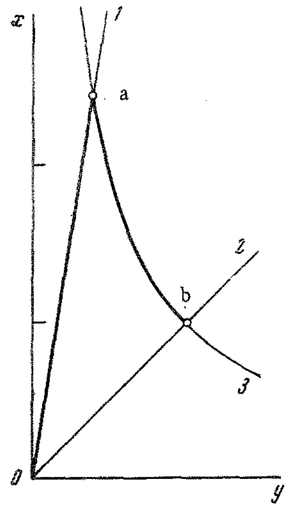

Fig. 2

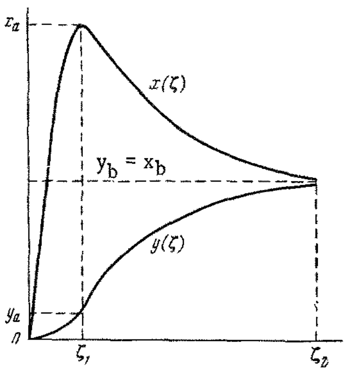

Fig. 3

For $\zeta \to +0$, the second term on the right side goes to zero, and

$$x_0 + {}^1/_2 x_0^2 = i_0, \quad \text{or} \quad x_0 = \sqrt{2i_0 + 1} - 1$$

To determine the value of y_0, we note that in the second equation of (1.5), just as in the first, the second term on the right side is insignificant at the time the source is effective; consequently, the variation in x and y at that time is described by

$$\frac{dx}{d\zeta} = \frac{i(\zeta)}{1+x}, \qquad \frac{dy}{d\zeta} = \lambda \frac{i(\zeta)}{1+x}$$

Dividing one equation by the other and integrating, we have

$$y(\zeta) = \lambda x(\zeta)$$

Passing to the limit $\zeta \to +0$, we have

$$y_0 = \lambda x_0$$

For $\zeta > 0$, we obtain from the second equation of (1.5) (in this case, the value of y_0 serves as an initial condition)

$$y(\zeta) = y_0 [1 + \lambda y_0 \zeta]^{-1}$$

after which, there follows from the first equation of (1.5)

$$x(\zeta) = x_0 [1 + \lambda y_0 \zeta]^{-1/\lambda}$$

3. We consider the variation in x and y in the case of an η source, i.e., for

$$i(\zeta) = \begin{cases} 0 & (\zeta < 0) \\ i_1 & (\zeta > 0) \end{cases}$$

In the case of an η source, Eqs. (1.5) are not analytically solvable. It is therefore advisable to carry out a qualitative study of the behavior of the solutions. We consider the "phase trajectory" of the system in the xy plane. An equation for it is obtained by dividing the first equation of the system (1.5) by the second:

$$\frac{dx}{dy} = \frac{1}{\lambda} \frac{i_1 - xy(1+x)}{i_1 - y^2(1+x)} \tag{3.1}$$

For $i_1 \gg 1$, $x \gg 1$ over a large portion of the phase trajectory, and Eq. (3.1) can be simplified to

$$\frac{dx}{dy} = \frac{1}{\lambda} \frac{i_1 - x^2 y}{i_1 - xy^2} \tag{3.2}$$

For small y, the conditions $x^2 y \ll i_1$ and $xy^2 \ll i_1$ are satisfied, and we obtain $x \approx y/\lambda$ from Eq. (3.2). Actually, $\lambda \ll 1$, and therefore $x \gg y$. With an increase in ζ, x increases, and both terms in the numerator of Eq. (3.2) are comparable.

In this case, however, the second term in the denominator is considerably less than the first. Therefore, over some portion of the phase trajectory, Eq. (3.2) can be replaced by the simpler equation

$$\partial x / \partial y = (i_1 - x^2 y) / \lambda i_1 \tag{3.3}$$

The substitution $x = i_1 \lambda z_y' / yz$ reduces Eq. (3.3) to the form

$$y z_{yy}'' - z_y' - y^2 z / i_1 \lambda^2 = 0$$

The general solution of the latter equation is

$$z = y\,[C_1 I_{2/3}(\xi) + C_2 I_{-2/3}(\xi)] \quad \left(\xi = \frac{2}{3}\frac{y^{3/2}}{\lambda i_1^{1/2}}\right)$$

where I_p is a Bessel function of purely imaginary argument and order p. Consequently,

$$x = \left(\frac{i_1}{y}\right)^{1/2} \frac{C_2 I_{1/3}(\xi) + C_1 I_{-1/3}(\xi)}{C_1 I_{2/3}(\xi) + C_2 I_{-2/3}(\xi)}$$

Since $x \to C_1/C_2$ when $y \to 0$, it is necessary to set $C_1 = 0$ in order to satisfy the zero initial conditions. We finally have

$$x = \left(\frac{i_1}{y}\right)^{1/2} \frac{I_{1/3}(\xi)}{I_{2/3}(\xi)} \tag{3.4}$$

The x(y) dependence given by Eq. (3.4) is shown in Fig. 1 ($\lambda = 10^{-3}$, $i_1 = 10^2$). A direct calculation shows that the neglect of the xy^2 term in the denominator of Eq. (3.2) is valid up to $y \sim 1$, where the argument of the Bessel function is considerably greater than one, and the asymptotic expression $x \approx (i_1/y)^{1/2}$ can be used for x. It is easy to see that the asymptotic behavior of x(y) corresponds to the "quasistationary" approximation which is obtained by setting the derivative of x equal to zero in Eq. (3.3). Therefore the phase trajectory of the system described by Eq. (3.2) can be approximated by the segments

$$x = \begin{cases} y/\lambda & \text{for} \quad 0 < y^3/\lambda^2 < i_1 \\ (i_1/y)^{1/2} & \text{for} \quad i_1 < y^3/\lambda^2 < i_1/\lambda^2 \end{cases}$$

The approximating curves are shown in Fig. 1 by the dashed lines.

Analysis of the equations for $i_1 \ll 1$ indicates that a similar approximation to the phase trajectory is possible in that case also. In an analysis of the behavior of the solutions of system (1.5) one can therefore assume that the phase trajectory has the form sketched in Fig. 2 (curve oab). In this figure, the curves 1 and 2 are described by the equations

$$x = y / \lambda, \quad y = x$$

respectively, and line 3 by the equation

$$y = i_1 / x(1 + x)$$

The points a and b have coordinates defined by the equations

$$x_a^2 (1 + x_a) = i_1 / \lambda, \quad y_a = \lambda x_a; \qquad x_b^2 (1 + x_b) = i_1, \quad y_b = x_b$$

The point b is a stationary point of the system (1.5).

A typical form of the relationships $x = x(\zeta)$, $y = y(\zeta)$, obtained from the qualitative analysis of the system (3.1) given above, is shown in Fig. 3.

We estimate the characteristic (dimensionless) times ζ_1 and ζ_2. From the first equation in (1.5) there directly follows

$$\zeta = \int_0^x \frac{(1+x)\,dx}{i_1 - x(1+x)\,y}$$

For $\zeta < \zeta_1$, one can set $y = \lambda x$, and in addition $x \gg 1$ over the greater portion of the range of integration when $i \gg 1$, so that

$$\zeta \approx \int_0^x \frac{x\,dx}{i_1 - \lambda x^3}$$

Making the substitution $x = (i_1/\lambda)^{1/3}u$ in the integral, we obtain

$$\zeta \approx \frac{1}{\lambda^{2/3} i_1^{1/3}} \int_0^u \frac{u\,du}{1-u^3}$$

There then follows directly

$$\zeta_1 \sim (\lambda^2 i_1)^{-1/3}$$

Similarly, we obtain from the second equation in (1.5)

$$\zeta = \frac{1}{\lambda} \int_0^y \frac{(1+x)\,dy}{i_1 - (1+x)y^2}$$

For $\zeta_1 < \zeta < \zeta_2$, a considerable portion of the range of integration corresponds to the portion ab of the phase trajectory (Fig. 2).

When $i_1 \gg 1$ on this portion, $x = (i_1/y)^{1/2}$, so that

$$\zeta = \frac{1}{\lambda i_1^{-1/3}} \int_0^{u_1} \frac{du}{\sqrt{u} - u^2}, \qquad u_1 = \frac{y}{i_1^{1/3}}$$

There then follows

$$\zeta_2 \sim \lambda^{-1} i_1^{-1/3}$$

Evaluations of the quantities ζ_1 and ζ_2 for $i \ll 1$ are obtained similarly. The results are

$$\zeta_1 \sim (i_1\lambda)^{-1/2}, \quad \zeta_2 \sim (\lambda^2 i_1)^{-1/2}$$

5. The analysis made of system of equations (1.5) leads to the conclusion that the variation in the electric field at the initial period of time ($\zeta < \zeta_1$) is mainly determined by the buildup of polarization charges during passage of Compton currents and the conductivity of the air has no effect on the field because of the low electron density.

When $\zeta > \zeta_1$, the field is determined by the equality of the Compton current and the conductivity current. In this time period ($\zeta < \zeta_2$), the electron density builds up because of the continuously acting ionization source. This leads to a rise in conductivity and therefore to a drop in the intensity of the electric field. Subsequently, the rise in electron density ceases because of electron-ion recombination, and the system becomes stationary when $\zeta > \zeta_2$.

Thus the quantity $\zeta = \zeta_1$ can be called the relaxation time of the electric field, and the quantity ζ_2 the relaxation time of the electron density. A comparison of estimates of the quantities ζ_1 and ζ_2 shows that $\zeta_2 \gg \zeta_1$ always, i.e., the electron density relaxes very much more slowly than the electric field.

The spatial asymmetry of a system leads to emission of a part of the energy in the form of an electromagnetic pulse. However, an analysis of the numerical solutions of such systems shows that inclusion of the emission introduces little change in the intensity and time behavior of the fields in the immediate area of the currents. In a spatially isotropic system a transverse (with respect to the γ-ray flux) field is absent, and the results obtained above are correct in this sense.

In conclusion, the authors thank G. M. Gandel'man for several discussions.

LITERATURE CITED

1. W. J. Karzas and R. Latter, "Electromagnetic radiation from a nuclear explosion in space," Phys. Rev., 126, No. 6 (1962).
2. A. S. Kompaneets, "Radio emission from an atomic explosion," Zh. Éksp. Teor. Fiz., 35, No. 6 (1958).
3. V. Gilinsky, "Kompaneets model for radio emission from a nuclear explosion," Phys. Rev., 137, No. 1A (1965).